15.3 Using coroutines to describe algorithms

Description

In this section, we show examples of how to use coroutines to describe a certain class of algorithms often referred to as *interlocked sequential execution stacks*.

A *generator* is a coroutine capable of producing a sequence of values. A new value is produced for each invocation of the coroutine. The next value depends on the sequence of previously generated values. We show how to define generators for computing the mathematical functions factorial. The main purpose of the factorial example is to illustrate how coroutines work.

We assume the reader is familiar with the factorial function, but here is a short description: The factorial of a non-negative integer N, denoted by N!, is the product of all positive integers less than or equal to N. First we show how to define factorial as a simple method:

```
FactorialFunction(N: var integer) -> F: var integer:
   F := N
   loop: do
      if (N > 1) :then
         N := N - 1
         F := F * N
         restart(loop)
```

As can be seen, `FactorialFunction` computes the product *N * (N – 1) * (N – 2) * … * 2 * 1*.

Next we show how to define a generator that produce the sequence of factorial, *1!, 2!, 3!, …, N!, …*. When the coroutine is resumed, it returns the next value in the sequence when it suspends.

```
PlainFactorial: obj
   getNext -> R: var integer:
      PlainFactorial.call
      R := F
   F: var integer
   N: var integer
   F := 1
   cycle
      PlainFactorial.suspend
      N := N + 1
      F := F * N
```

When `PlainFactorial` is generated it executes statements until the first execution of `PlainFactorial.suspend`. An invocation of ++++

Explanation, perhaps OSD showing the recursion. `PlainFactorial.getNext` will ~~then~~ return the factorial of 1 which is 1. Subsequent invocations will return the next factorial as shown below:

```
UsingPlainFactorial: obj
   PlainFactorial: obj
      -"-
   V: var integer
   V :=  -- V = 1
   V :=  -- V = 2
   V :=  -- V = 6
   V :=  -- V = 24
```

The next figure shows a snapshot of the execution of `UsingPlainFactorial` where:

- `PlainFactorial` has been generated and suspended execution during the first iteration of `cycle`.

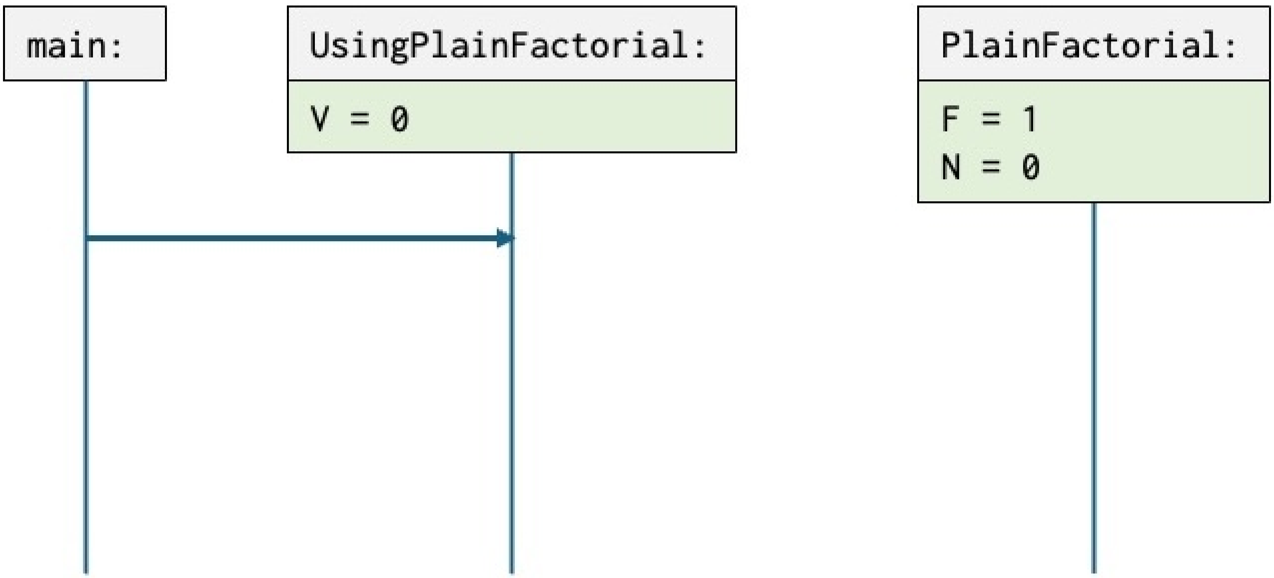
- The read arrow –> shows that `PlainFactorial` is suspended just before the statement `N := N + 1`.
- The point of execution in `UsingPlainFactorial` is before the first statement `V := PlainFactorial.next` as indicated by the read arrow ==>.
- A read arrow of the form ==> shows the active point of execution.
- A read arrow of the form –> show the point of suspension of a coroutine.

The OSD in the right column illustrates that `UsingPlainFactorial` is currently executing and that `PlainFactorial` is suspended.

+++ Skal der forklares hvordan man kan se at den er suspended?

Den er ikke med i sekvensen som starter i main, og man ved jo at den er genereret, men det er vel ikke nok? OLM: måske nok!? Hvad kan den ellers være?

```
UsingPlainFactorial: obj
   PlainFactorial: obj
      -"-
      F := 1
      cycle
         PlainFactorial.suspend
-->      N := N + 1
         F := F * N
    V: var integer
==> V :=
    V :=
    V :=
    V :=
```

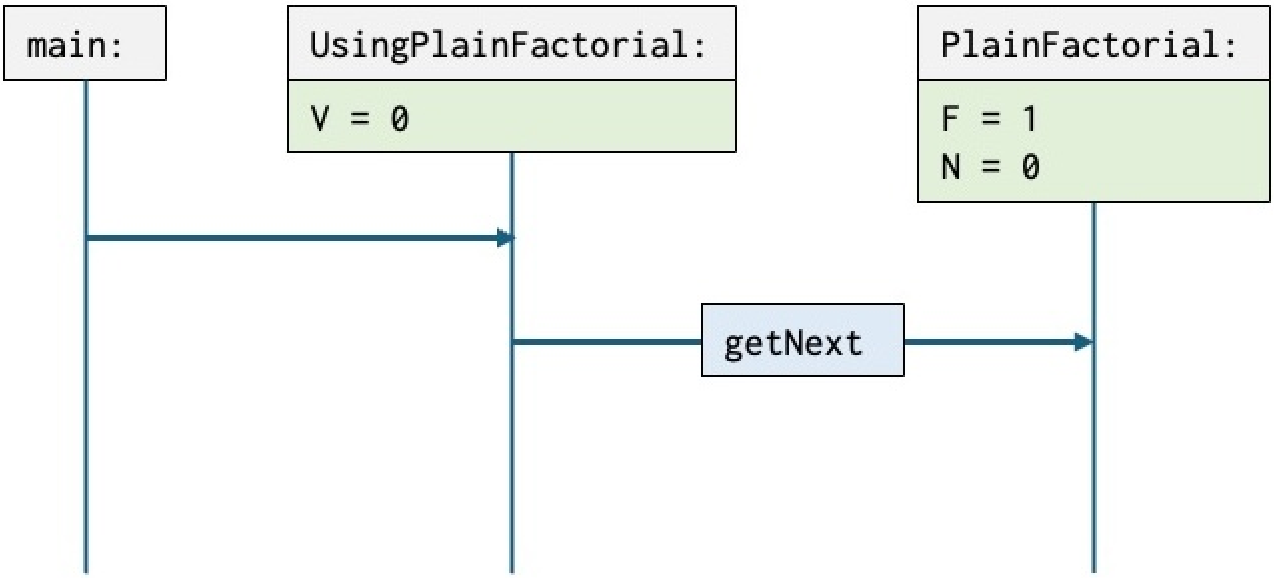


The OSD also shows the values of the variable `V` in `UsingPlainFactorial`, and the variables , `F` and `N` in `PlainFactorial`.

The next next scenario shows the situation:

- `PlainFactorial.getNext` has been executed by `UsingPlainFactorial`.
- `getNext` has executed `PlainFactorial.call`.
- The next statement to be executed is `N := N + 1`.

```
UsingPlainFactorial: obj
   PlainFactorial: obj
      getNext -> R: var integer:
         PlainFactorial.call
         R := F
      F: var integer
      N: var integer
      F := 1
      cycle
         PlainFactorial.suspend
==>      N := N + 1
         F := F * N
   V: var integer
   V :=
   V :=
   V :=
   V :=
```

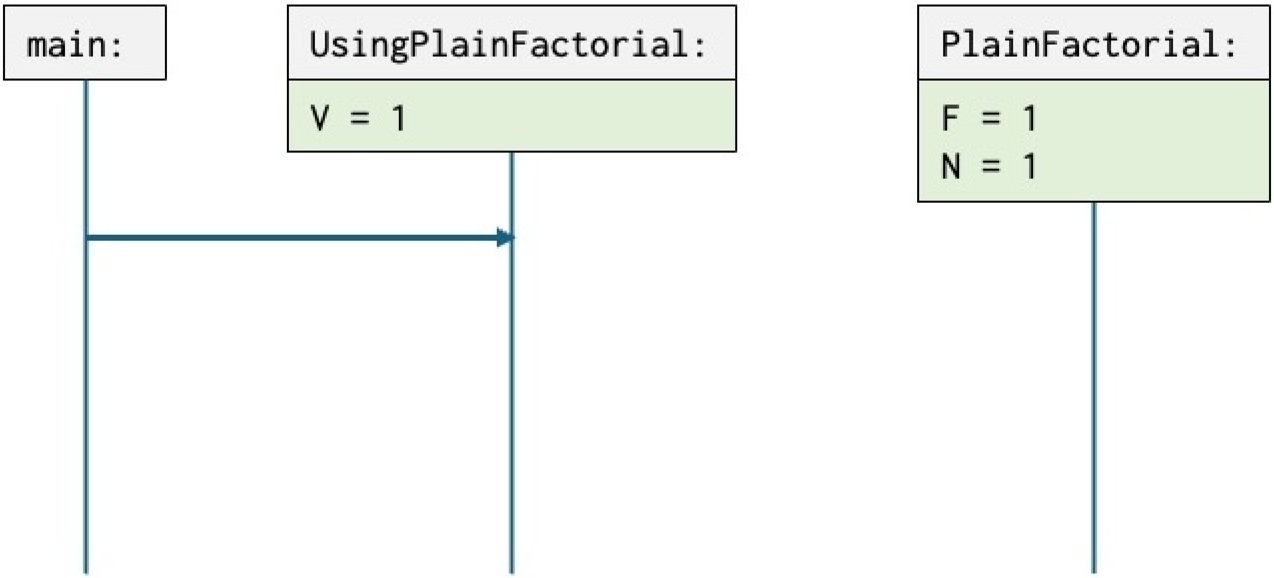


The next snapshot shows the situation after `PlainFactorial` has executed the two assignments statements, `N := N + 1` and `F := F * N`, and executed a `PlainFactorial.suspend` during the second iteration of `cycle`:

- `PlainFactorial` is suspended before the statement `N := N + 1` – as in the first figure above.
- The active point of execution is in `UsingPlainFactorial` before the second statement `V := PlainFactorial.getNext`.
- The variables have been updated `V`, `F` and `N` have all new values.

```
UsingPlainFactorial: obj
   PlainFactorial: obj
      -"-
      cycle
         PlainFactorial.suspend
-->      N := N + 1
         F := F * N
    V: var integer
    V :=
==> V :=
    V :=
```

```
    V :=
```

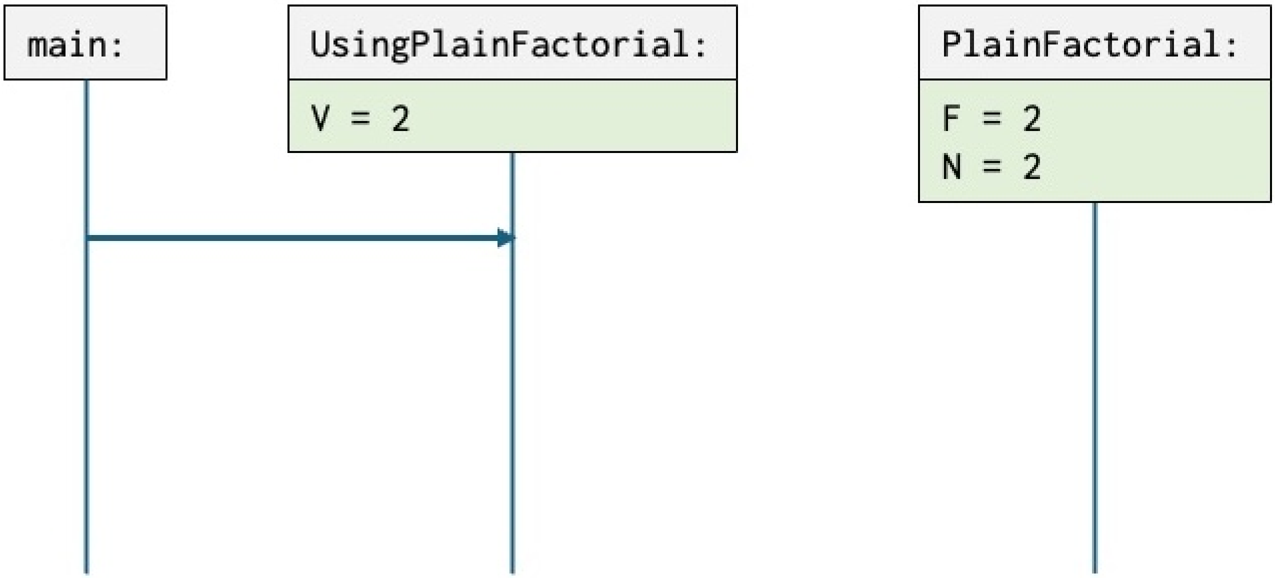


Execution of the second `V :=` resumes execution of `PlainFactorial`, which then generate the next factorial value. We don't show a snapshot of this.

The next snapshot shows the situation after `PlainFactorial` has suspended:

- `PlainFactorial` is as in previous snapshots suspended before the statement `N := N + 1`.
- The current point of execution is in `UsingPlainFactorial` before the third statement `V := PlainFactorial.getNext`.
- The variables `V`, `F` and `N` have been updated.

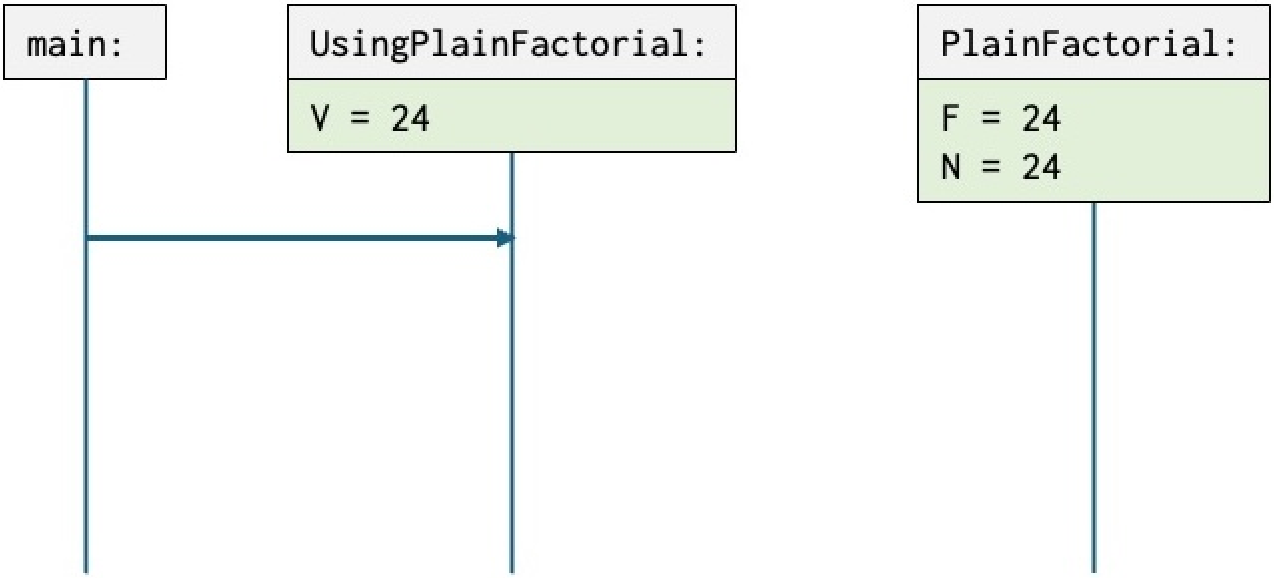
```
UsingPlainFactorial: obj
   PlainFactorial: obj
      -"-
      cycle
         PlainFactorial.suspend
-->      N := N + 1
         F := F * N
    V: var integer
    V :=
    V :=
==> V :=
    V :=
```



Finally we show a snapshot at the situation after execution of the last `V := PlainFactorial.getNext`:

- `PlainFactorial` is (again) suspended before the statement `N := N + 1`.
- The active point of execution is after the last statement.
- The variables `V`, `F` and `N` have been updated.
- `V` = 24 = 6!

```
UsingPlainFactorial: obj
   PlainFactorial: obj
      -"-
      cycle
         PlainFactorial.suspend
-->      N := N + 1
         F := F * N
    V: var integer
    V :=
    V :=
    V :=
    V :=
==>
```



Burde måske stå noget om hvordan activities terminerer (rigtig ord?); det samme som ville sige når toppen på stakken terminerer.

Recursive factorial generator

Kommentarerne om main ovenfor gælder selvfølgelig også her, og hvordan man beskriver at at program udføres.

Next we show a a version of a factorial generator using a recursive method instead of the loop implemented using `cycle`. As said the purpose is to show how coroutines work and not necessarily a recommended programming style.

```
RecursiveFactorial: obj
    getNext -> R: var integer:
       RecursiveFactorial.call
       R := F
    F: var integer
    N: var integer
    next:
       RecursiveFactorial.suspend
       N := N + 1
       F := F * N
       next
    F := 1
    next
```

When `RecursiveFactorial` is generated it invokes the local method `next`, which suspends execution. Successive invocations of `next` resumes the coroutine and returns the~~n~~ `next` factorial (F) in the sequence.

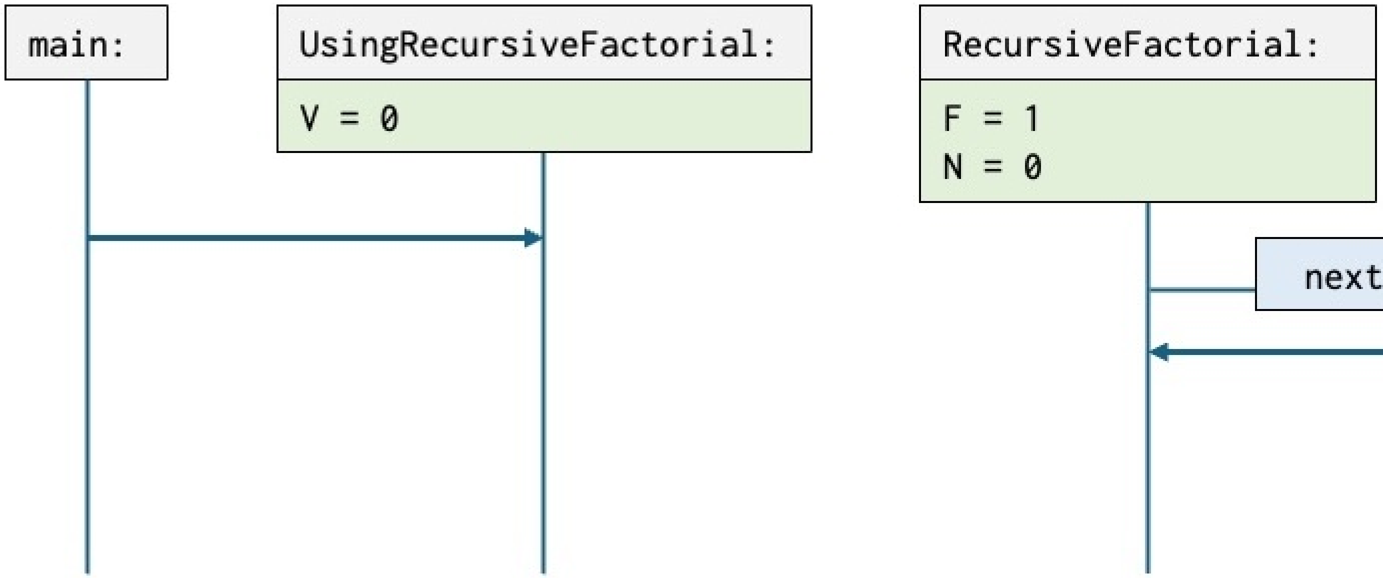
The next figures shows snapshots of using `RecursiveFactorial` by the object `UsingRecursiveFactorial`:

```
UsingRecursiveFactorial: obj
   RecursiveFactorial: obj
      -"-
   V: var integer
   V := Recursive -- V = 1
   V := Recursive -- V = 2
   V := Recursive -- V = 6
   V := Recursive -- V = 24
```

The first snapshot how the situation after the objects `UsingRecursiveFactorial` and `RecursiveFactorial` have been generated:

- `RecursiveFactorial` is suspended before `N := N + 1` in the method object `next`.
- Note that `RecursiveFactorial` has been suspended while execution an instance of the method `next`.
- The active point of execution is at the first `V := RecursiveFactorial.getNext` in `UsingRecursiveFactorial`.
- The values of the variables are `V = 0`, `F = 1` and `N = 0`.

```
 UsingRecursiveFactorial: obj
    RecursiveFactorial: obj
       getNext -> R: var integer:
          RecursiveFactorial.call
          R := F
       F: var integer
       N: var integer
       next:
          RecursiveFactorial.suspend
-->       N := N + 1
          F := F * N
          next
       F := 1
       next
    V: var integer
==> V := Recursive
    V := Recursive
    V := Recursive
    V := Recursive
```

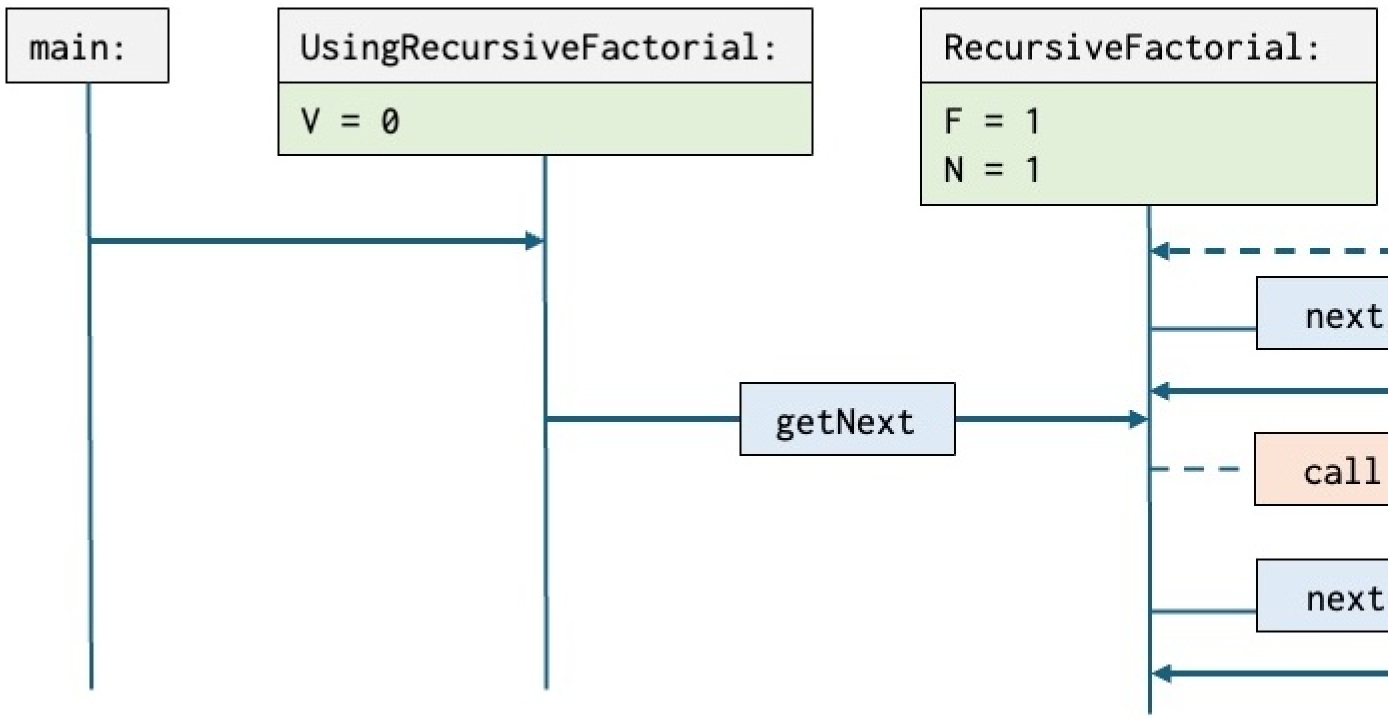


Her er det vel vigtig at få formidlet at RecursiveFactorial har en activity som bliver udført ved hver RecursiveFactorial.call.

The next snapshot shows a situation after execution of the first `RecursiveFactorial.getNext`:

- `RecursiveFactorial` has been resumed.
- `N := N + 1` has been executed — `N = 1`.
- `F := F * N` has been executed — `F = 1`.
- A recursive invocation of `next` has been generated.
- The active point of execution is at `RecursiveFactorial.suspend` in this `next` method.

```
 UsingRecursiveFactorial: obj
    RecursiveFactorial: obj
       -"-
       next:
==>       RecursiveFactorial.suspend
          N := N + 1
          F := F * N
          next
       F := 1
       next
    V: var integer
    V := Recursive
    V := Recursive
    V := Recursive
    V := Recursive
```



Har nu indsat en call i figuren med en speciel markering på pilen og farven
Det er ikke oplagt at metodekaldet med `getNext` skal pege som den nu gør på figuren.
Rækkefølgen af kaldene er fortsat korrekt.
Men stakken er stadig mudret.
Og der skal faktisk også en call ind på flere af de andre figurer hvis vi skal være konsistente!

call-pilen til et punkt før det første kald på kan nemt misforstås til at man da kalder den samme next igen, selvom jeg godt ved at pilen fra call ikke skal forstås sådan. Skal man først have denne notation for metodekald, så ville det næsten være bedre at call-pilen træffer lifeline under call.

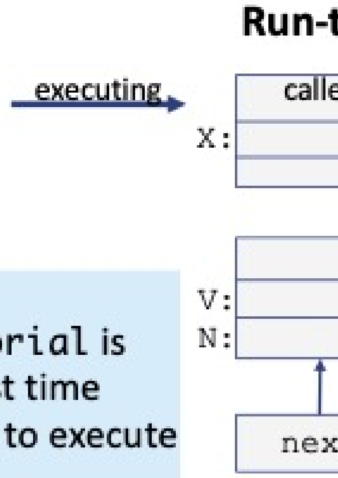
Man skal vel kunne generere stakken ved at følge pilene, f.eks. …, getNext, next, next.

Synes dette bliver mere tydeligt med slide 14 i Coroutines-book.pptx! Også call-pilen.
OLM: lad os tage snakken ud fra denne – dem der pt er her skal laves om.

```
RecFactorial: obj
      out V: var integer
      N: var integer
      next:
         RecFactorial.suspend
         N := N + 1
         V := V * N
         next
      V := 1
      next

X: var integer
X := RecFactorial -- X = 1
X := RecFactorial -- X = 2
X := RecFactorial -- X = 6
X := RecFactorial -- X = 24
```

- Initial state
- RecursiveFactorial is suspended the first time
- The caller is ready to execute first call of RecursiveFactorial

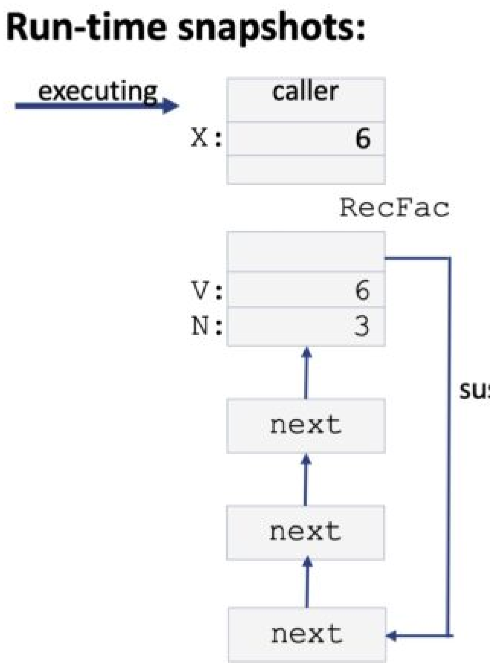



```
V := Recursive -- V = 2

V := Recursive -- V = 6

V := Recursive -- V = 24
```

```
RecFactorial: obj
      out V: var integer
      N: var integer
      next:
         RecFactorial.suspend
         N := N + 1
         V := V * N
         next
      V := 1
      next
X: var integer
X := RecFactorial -- X = 1
X := RecFactorial -- X = 2
X := RecFactorial -- X = 6
X := RecFactorial -- X = 24
```



Plus more not shown

As can be seen, a `suspend` within `next`, suspends the whole execution stack and a `call` resumes execution of the top element of the stack. +++ Bedre formulering.

Merging binary search trees

The next example is more interesting. Here we show how to merge two binary search trees. We assume that the reader is familiar with the concept of a binary search tree +++ evt henvisning.

In this example, we define a binary tree where the a node contains a `String` being the name of a person. First we define class `BinarySearchTree`:

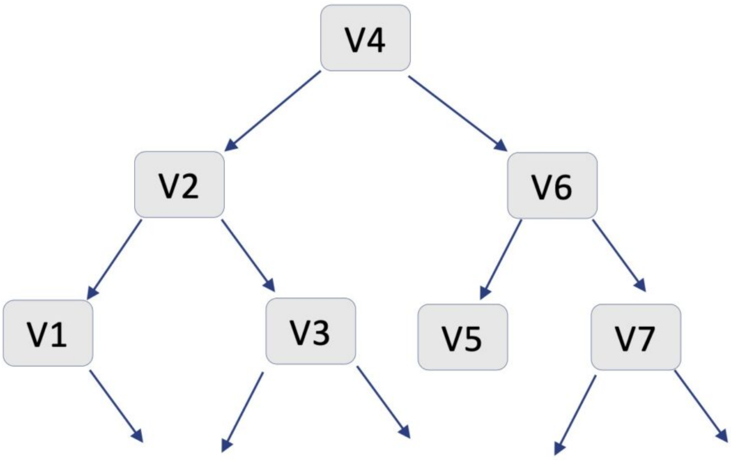
```
class BinaryTree:
   class node(elm: var String):
      left: ref node
      right: ref node
      insert(S: var String):
         if (S <= elm) :then
            if (left == none) :then
               left := node(S)
            :else
               left.insert(S)
         :else
            if (right == none) :then
               right := node(S)
            :else
               right.insert(S)
      print(ind: var integer):
        ...
   root: ref node
   insert(S: var string):
      if (root == none) :then
         root := node(S)
      :else
         root.insert(S)
   :::
```

A `BinaryTree` has the following attributes:

1. A local class `Node`.
2. A reference `root`, that refers to the root (top `Node`) of the tree.
3. A method `insert` for inserting a `Node` in the tree with the parameter `S` being the `String` stored in the `Node`. It works as follows:
    - If `root == none`, then the insertion is the first `Node` else `root.insert(S)` is invoked.
4. A `Node` has the following attributes:
5. References `left` and `right` that refer to the left and right branches of the `Node` respectively.
6. An `insert` method that works as follows:
    - If `S <= elm` where `elm` is the name in the current `Node`, then `S` is inserted in the left branch.
        - If `left == none` then `S` is the first element in the left branch and `left := Node(S)` is executed otherwise `left.insert(S)` is executed recursively.
    - If `S > elm` then `S` is inserted in the right branch.
7. A print `method` not shown.

When a `Node` is inserted into the tree it is ordered based on a lexicographical ordering. I.e. “Dave” comes before “John” (“Dave” < “John”).

Preliminary figure



We may then declare two `BinaryTree` objects and insert some elements into them:

```
boys: obj BinaryTree
girls: obj BinaryTree
boys.insert("Peter")
girls.insert("Cecilie")
girls.insert("Maria")
boys.insert("Robin")
...
```

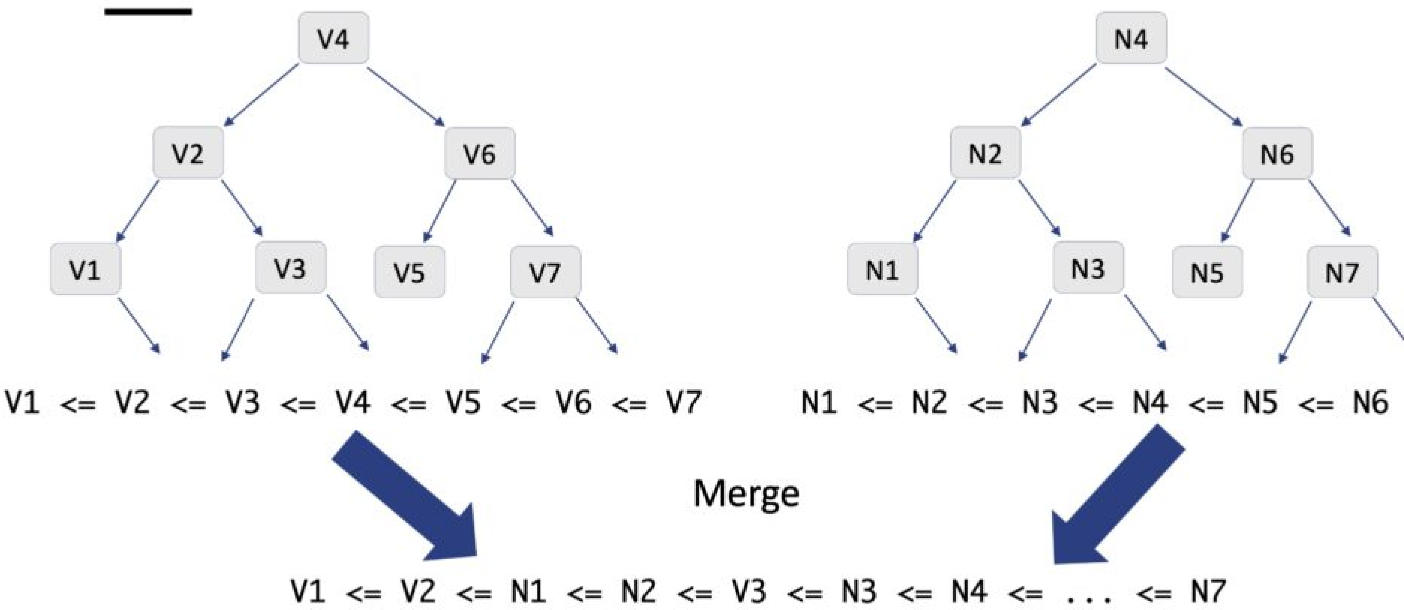
Next we add a coroutine, `theScanner`, that traverse the tree and for each node in the tree, it suspends execution and returns the value at the node. If the tree has n nodes it returns a sequence of `String` values where `V1 <= V2 <= V3 <= ... <= Vn.`

```
   class BinaryTree:
      -"-
      theScanner: obj   +++ scanner
         CV: var String
         next -> V: var String:
            theScanner.call
            V := CV
         scan(current: ref node):
            if (current =/= none) :then
               scan(current.left)
               CV := current.elm
               theScanner.suspend
               scan(current.right)
         theScanner.suspend
         scan(root)
         CV := ""
         theScanner.suspend
      next -> V: var String:
         V := theScanner.next
```

Finally we may add a method, `merge` that merges the values of two binary search trees:

```
   merge:
      nextBoy: var String
      nextGirl: var String
      tail(T: ref BinaryTree):
         cycle
            S: var String
            S := T.next
```

```
         if (S = "") :then
            leave(tail)
         :else
            S.print
   nextBoy := boys.next
   nextGirl := girls.next
   loop: do
      if (nextBoy <= nextGirl) :then
         if (nextBoy = "") :then
            nextGirl.print
            tail(girls)
         :else
            nextBoy.print
            nextBoy := boys.next
            restart(loop)
      :else
         -- nextBoy > nextGirl
         if (nextGirl = "") :then
            nextBoy.print
            tail(boys)
         :else
            nextGirl.print
            if (nextBoy.length > 0) :then
               nextGirl := girls.next
            restart(loop)
```



The merge method works ass follows:

1. The statements: `nextBoy := boys.next` and `nextGirl := girls.next` assigns the first boy and first girl to `nextBoy` and `nextGirl` respectively where the ordering is alphabetical.
2. If `nextBoy <= nextGirl`, then `nextBoy` is printed and `nextBoy` is assigned the next boy from the tree using `boys.next`.
3. If `nextBoy > nextGirl`, then `nextGirl` is printed and assigned the next girl.
4. This is repeated until no more boys and girls in the trees.
5. The termination condition is that the empty string (`""`) is returned by `next` if no more girls/boys in a tree.
6. if the boys tree becomes empty before the girls tree, then the method invocation `tail(girls)` prints the remaining girls in the tree – similarly if the girls tree becomes empty before the boys tree.